International Journal of advancement in electronics and computer engineering (IJAECE)
Volume 1, Issue 6, Sep 2012, pp.147-151, ***ISSN 2278 -1412***

A Novel Approach of Image Denoising Based On OWT Using Dual Tree Domain

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Abstract *– Image denoising is a common procedure to suppress the quality degradation caused by noise. Several image denoising methods are proposed in literature. Amongst these Discrete Wavelet Transform-DWT Filters are very popular. Denoising using the DWT-Transform includes decomposition of the image into various sub bands and then modeling them as independent identically distributed random variables with Gaussian distribution. Shrinkage methods are often used for suppressing Additive White Gaussian Noise (AWGN), where thresholding is used to retain the larger wavelet coefficients alone. Minimum Mean Square Error estimation is a common practice for noise analysis and is thus included in this paper. Overall we discuss the denoising using Orthonormal transform using dual tree domain.*

Keywords*: Wavelet transform, Orthonormal transform, SURE, Dual Tree Transform*

I. Introduction

Image denoising algorithms are often used to enhance the quality of the images by suppressing the noise level while preserving the significant aspects of interest in the image. Several methods are proposed in literature for image denoising, where Discrete Wavelet Transform-Domain filters are very popular. A wavelet transform is the representation of a function by wavelets. Here we propose a new estimator for image denoising a 2D dual tree M-band wavelet transform. Our approach relies on an extension of Stein's formula which allows us to take into account the specific correlations of the noise component. The objective of this paper is to develop a novel approach of image denoising based on OWT (Orthonormal Wavelet Transform) using Dual Tree Domain.

II. Wavelet Transform

A Wavelet is a "small wave", which has its energy concentrated in time. It gives a tool for the analysis of transient, non stationary, or time-varying phenomenon. It not only has an ability to allow simultaneous time and frequency analysis with a flexible mathematical foundation. A wavelet transform is classified into continuous wavelet transform (CWT) and DWT .The continuous wavelet transform (CWT) has received significant attention for its ability to perform a time-scale analysis of signals. On the other hand, the discrete wavelet transform (DWT) is an implementation of the wavelet transform using a discrete set of wavelet scales and translations obeying some definite rules. In other words, this transform decomposes the signals into mutually orthogonal set of wavelets. In this work we only discuss methods based on DWT. Calculating wavelet coefficients at every possible scale is a fair amount of work, and it generates an awful lot of data. What if we choose only a subset of scales and positions at which to make our calculations? It turns out rather remarkably that if we choose scales and positions based on powers of two so called dyadic scales and position then our analysis will be much more efficient and just as accurate. We obtain such an analysis from the discrete wavelet transform (DWT)[1].

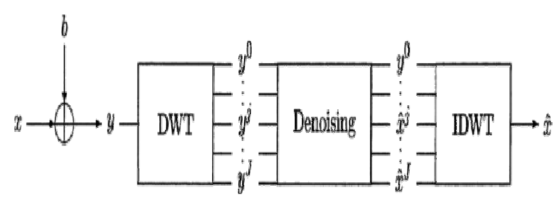

Fig.1. Principle of Wavelet denoising

II.1. Orthonormal Transform

The Orthonormal wavelet transform is an efficient way for signal representation since there is no redundancy in its expression [2,3], but due to aliasing in the decimation stage it lacks the often desired property of shift invariance.

II.2. SURE

SureShrink [4,5] is an adaptive thresholding method where the wavelet coefficients are treated in level-by-level fashion. In each level, when there is information that the wavelet representation of that level is not sparse, a threshold that minimizes Stein's unbiased risk estimate (SURE) is applied. Sure Shrink is used for suppression of additive noise in wavelet-domain where a threshold T_{SURE} is employed for denoising. The threshold parameter T_{SURE} is expressed as:

$T_{sure} = \arg\min_{T_k} (SURE(T_h; Y))$

$(SURE(T_h; Y))$ is define by:

$$SURE(T_h; Y) = \sigma^2 - \frac{1}{L} \times \left[2\sigma_n^2, \left\{ i : |Y_i| \le T_h - \sum_{i=1}^{L} \min(|Y_i|, T_h)^2 \right\} \right]$$

Where,
σ^2_n is the noise variance of AWGN;
L is the total number of coefficients in a particular sub-band;
Y_i is a wavelet coefficient in the particular sub-band.

II.3 Dual Tree Transform

Wavelet shrinkage has become an efficient method for image denoising. It consists in projecting discrete data onto a basis (usually an Orthonormal one) and applying a nonlinear operator to the transformed coefficients. A simple thresholding rule is often used as a nonlinear estimator. The denoised signal is then recovered by the inverse transform. Discrete wavelet transforms (DWT) possess good decorrelation properties and provide sparse representations for a variety of regular images. For the Visushrink and the SureShrink methods, Donoho et al. have derived optimal scalar Thresholds[1]. Many improvements on scalar thresholding have been investigated subsequently, such as block-thresholding which accounts for local dependence between neighboring wavelet coefficients [2]. From the transform choice viewpoint, the DWT is maximally decimated, which hampers its robustness to signal shifts. Undecimated wavelets or more general over complete expansions have thus been proposed to alleviate some of the wavelet decomposition Shortcomings. However, frame decompositions introduce correlations between the signal/noise components which have to be taken into account in the design of the regression rule. The dual-tree discrete wavelet transform [6,7] is one of the most promising frame decompositions due to its reasonable computational cost, limited redundancy and improved selectivity features for image applications. This decomposition is based on two classical DWT operating in parallel, employing a Hilbert pair of mother wavelets. In our recent work [8], we have proposed-band extensions to the dual-tree transform (DTT) and investigated their properties. We have proposed-band extensions to the dual-tree transform (DTT) and investigated their properties. The objective of this research is to build a reliable estimator in the M-band DTT domain. The novelty of our approach consists in both analyzing the statistical properties of the noise coefficients.

III. Estimator Based On OWT Using DTD

III.1. Stein's formula

Noise statistics will be exploited to derive an adaptive estimator. To this purpose, we firstly state an extended form of Stein's principle [9] which will be useful in the next subsection. Hereafter, the considered random vectors are assumed to be real-valued**.**

Proposition 1
Let b ∈ n, b > 1, and

$$\bar{r} = \bar{s} + \bar{n}$$

Where $\bar{n}$ a B-dimensional zero-mean Gaussian is random vector and $\bar{s}$ is a B-dimensional random vector which is independent of $\bar{n}$. These vectors are decomposed as.

$$\bar{r} = \begin{bmatrix} r \\ \tilde{r} \end{bmatrix}, \ \bar{s} = \begin{bmatrix} s \\ \tilde{s} \end{bmatrix}, \ \bar{n} = \begin{bmatrix} n \\ \tilde{n} \end{bmatrix}$$

Where r, s and n are scalar random variables. Let $T=R^B \rightarrow R$ be a continuous, almost everywhere differentiable function satisfying some technical requirements [9]. Then,

$$E[T(\bar{r})s] = E[T(\bar{r})r] - E[n^2]E\left[\frac{\partial T(\bar{r})}{\partial r}\right] - E\left[\frac{\partial T(\bar{r})}{\partial \tilde{r}^T}\right]E[\tilde{n}n]$$

III.2. Proposed Estimator

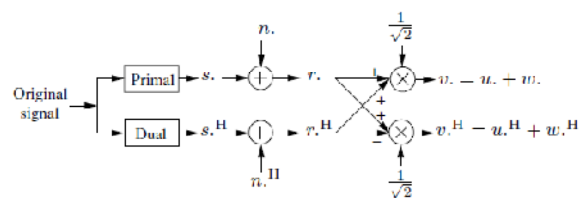

Fig. 2. Considered Model

As illustrated in Fig. we consider the following additive noise model in the DTT domain:

r. (k) = s. (k) +n. (k)

Where r.(k) is the wavelet coefficient of the observed noisy image at a given level j, in a given sub band m, at a spatial position k, similar notations being used for the

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original image and the noise. Inspired by previous works on block thresholding [2, 10], we are interested in applying the following shrinkage function:

$$\eta(\|\bar{r}.(k)\|) = \left(\frac{\|\bar{r}.(k)\|^{\beta.}-\lambda.}{\|\bar{r}.(k)\|^{\beta}}\right)_+$$

where λ. and β. are positive parameters, **r**.(**k**) is a vector containing the wavelet coefficients to be estimated and some possible neighbors. These neighboring values can be taken from the primal tree in the same sub band as in [2], or from the dual sub band as well. It is important to point out that η includes well known shrinkage rules as particular cases, for specific values of λ and β. We have already applied such a kind of shrinkage to denoise multi component images in the conventional DWT domain [11]. As two sets of coefficients are generated by a dual-tree transform, we aim at designing accurate estimators' ˆs. of s. and ˆsH. of sH having a common structure:

$$\hat{s}.(k) = \eta(\|\bar{r}.(k)\|)s.(k), \quad \widehat{s^H}.(k) = \eta^H(\|\bar{r}^H.(k)\|)s^H.(k)$$

where, for the dual tree, $\bar{r}^H(k)$ plays a role symmetric to $\bar{r}.(k)$The shrinkage function ηH has the same form as the η one, making use of parameters λH. and βH . Instead of λ. and β.. Here, the parameters λ. and λH. can be respectively considered as threshold values in the soft-thresholding of $\|\bar{r}.(k)\|^{\beta}$. and $\|\bar{r}.(k)\|^{\beta}$Since the primal and the dual tree play analogous roles, we only develop the theoretical results for the primal tree. Indeed, expressions concerning the dual tree are easily obtained by replacing any variable g (scalar, vector or matrix) by its dual counterpart gH. Our next objective is to find the threshold λ. and the exponent β. that minimize the quadratic risk R(λ., β.) = E[|s.(**k**) − ˆs.(**k**)|2]. The risk reads

$$R(\lambda,\beta) = E[S^2.(K)] + E\left[\left(\eta\|\bar{r}(k)\|r.(k)\right)^2\right] - 2E[\eta(\|\bar{r}.(k)\|)r.(k)s.(k)].$$

Unfortunately, as the wavelet coefficients s.(**k**) are unknown, it may appear impossible to calculate explicitly the last term in the expression of R(λ., β.). However, for an additive Gaussian noise, Prop. 1 applied to our estimator yields.

$$E[\eta(\|\bar{r}.(k)\|)r.(k)s.(k)] = E[\eta(\|\bar{r}.(k)\|)r^2.(k)] - \sigma^2 E\left[\frac{\partial(\eta(\|\bar{r}.(k)\|r.(k))}{\partial r.(k)}\right] - E\left[\frac{\partial(\eta(\|\bar{r}.(k)\|)r.(k))}{\partial \bar{r}^T(k)}\right]\boldsymbol{\Gamma}^{(\bar{n},n)}$$

where the vector r̄ (k) is decomposed as (‖r̄.(k)‖r.(k))and, using similar notations for the noise components, $\boldsymbol{\Gamma}^{(\bar{n},n)}\ E\left[\frac{\partial(\eta(\|\bar{r}.(k)\|)r.(k))}{\partial \bar{r}^T(k)}\right]$ From (13), we deduce after some calculations that:

(∂(η(□r □.(k) □)r.(k)))/(∂r. (k) = η(‖r̄.(k)‖)+□.ρ.(k)r.(k)

AND

∂(η(□r □.(k) □)r.(k)))/∂r̃. (k) = λ.ρ.(k)r.(k)

where

$$\rho.(k) = 1\left\{\|\bar{r}.(k)\|^{\beta.} > \lambda\right\}\frac{\beta}{\|\bar{r}.(k)\|^{\beta.+2}}r.(k).$$

III.3. Closed form expression of the risk

From the above calculations, the risk R (λ, β.) can be expressed as the expected value of

R (λ, β, **k**) = a. (k)λ^2 . + b. (k) λ. + c. (k)

where

$$a.(k) = 1\|\bar{r}.(k)\|^{\beta.} > \lambda\}\frac{r^2}{\|\bar{r}.(k)\|^{2\beta.}}$$

$$b.(k) = 2\sigma^2\left(\beta.r.(k)\frac{r.(k)+k(k)}{\|\bar{r}.(k)\|^{\beta.+2}} - \|\bar{r}.(k)\|^{-\beta.}\right)1\{\|\bar{r}.(k)\|^{\beta.} > \lambda\}$$

$$c.(k) = r^2.(k) - \sigma^2 + 1\{\|\bar{r}.(k)\|^{\beta.} > \lambda.\}(2\sigma^2 - r^2.(k))$$

$$k.(k) = -\sigma^2\check{r}^T.(k)\boldsymbol{\Gamma}^{(\bar{n},n)}$$

III.4 Computation of the parameters λ. and β.

Under mild conditions, R(λ., β.) is estimated by an empirical average ˆR (λ., β.) computed over the Kj observations in a given sub band at resolution j. The optimal values of λ. and β. are found according to a similar procedure to the one used to derive the SureShrink estimator. To this purpose, the observed variables ‖r̄(k)‖ are sorted in descending order, so that,

$$\|\bar{r}.(k_1)\| \geq \|\bar{r}.(k_2)\| \geq \cdots \geq \|\bar{r}.(k_{kj})\|.$$

for

$i_0\|\bar{r}.(k_{io-1})\|^{\beta} > . \geq \|\bar{r}.(k_{i0})\|^{\beta}$,
the risk estimate can be expressed as

$$R(\lambda,\beta) = \frac{1}{k_j}\left(\sum_{l=1}^{l_0-1} R(\lambda,\beta,k_l) + \sum_{l=l_0}^{k_j} R(\lambda,\beta,k_l)\right),$$

Or equivalently as

$$k_j\hat{R}(\lambda,\beta) = \boldsymbol{\lambda^2.\sum_{i=1}^{i_0-1}\frac{r^2.(k_i)}{\|\bar{r}.(k_i)\|^{2\beta.}} + 2\sigma^2\lambda.\sum_{i=1}^{i_0-1}\left(\beta.r.\frac{(k_i)(r.(k_i)+k.(k_i))}{\|\bar{r}.(k_i)\|^{2\beta+2.}} - \|\bar{r}.(k_i)\|^{-\beta.}\right) + \sum_{i=i_0}^{k} r^2.(k_i) - (K - 2i_0 + 2)\sigma^2}$$

For a given value of β, an optimal value λ* of λ. is obtained by minimizing the so-defined piecewise second-order binomial function. Then, a search on the optimal value of β. is carried out to minimize ˆR (λ, of β).

IV. Experimental Results

The various images are used for denoising which are representative set of standard 8-bit grayscale images such as LEENA, PEPPER AND EYE. All corrupted by simulated additive Gaussian white noise at five different power levels, minimum σ =0.001and maximum σ = 0.005 The denoising process has been performing with minimum variance of 0.001 , minimum threshold of 0 can be inserted up to maximum threshold of 100. In this process, we are sampled Orthonormal wavelet transform with eight resisting moments (sym8) over time decomposition stage. The Table 1 shows the PSNR values.

TABLE I

Image	Denoising Approach	Noisy Image		Denoised Image	
		MSE	**PSNR**	**MSE**	**PSNR**
Pepper	Normal	627.6	19.94	409.3	21.80
	SureShrink	625.0	19.94	316.9	22.91
	Modified Sure	627.2	19.94	199.9	24.91
Leena	Normal	636.8	18.88	434.0	20.55
	SureShrink	646.9	18.81	332.6	21.61
	Modified Sure	641.9	18.85	251.0	22.93
Eye	Normal	629.6	20.00	413.7	21.83
	SureShrink	630.1	19.99	311.7	23.05
	Modified Sure	626.7	20.02	190.0	25.21

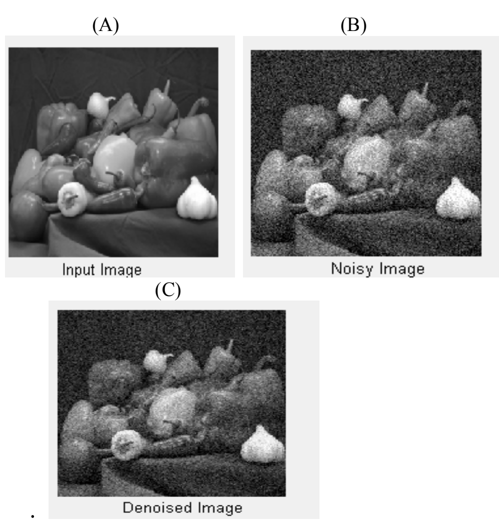

Fig.2. Normal image denoising

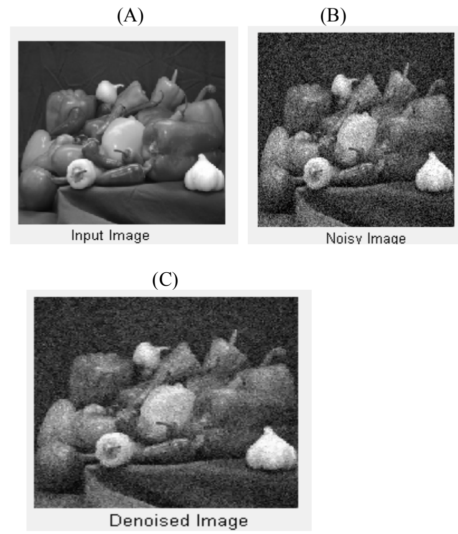

Fig.3 SureShrink denoising

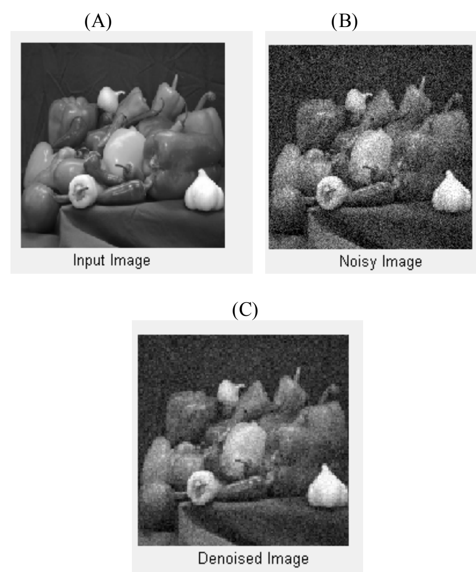

Fig.4. Modified SURE denoising

V. Conclusion

The Modified SURE approach is able to perform well when noise is Higher It gives Higher PSNR Value as compared to the Normal Image Denoising where Threshhold varies manually and SURE approach of image denoising where threshold is determined by SURE Estimator.

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